Towards Efficient and Verified Virtual Machines for Dynamic Languages

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The Success and Shortcoming of Virtual Machines

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- Performance > correctness & security

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- Dynamic code generation

Optimization of Verified Interpreters is a More Realistic Paradigm

- Small & simple $\rightarrow$ easy formalization

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- Small & simple $\rightarrow$ easy formalization
- Sufficient performance
- Balance between complexity and performance
- Promising experience with early implementation

Purely Interpretative Optimizations

Dyn Inca Ubx

Three languages

- **Dyn** Simple interpreter for dynamically typed languages
- **Inca** Type-based optimization (inline caching)
- **Ubx** Data-representation optimization (unboxing)

Purely Interpretative Optimizations

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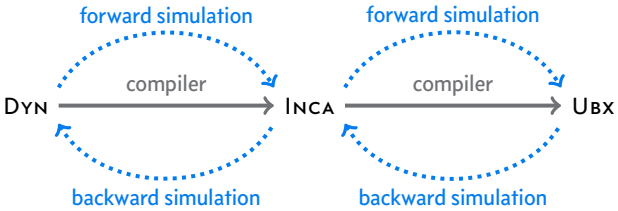

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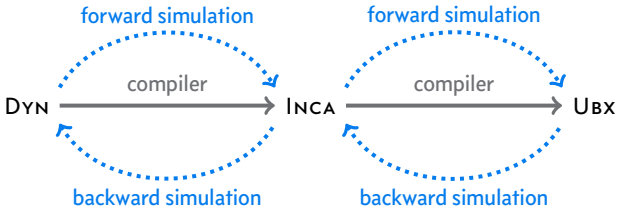

Three languages

- DYN Simple interpreter for dynamically typed languages
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Mechanized formalization

- Isabelle/HOL
- 4800 lines + 1000 background
- Extensive use of locales
- Realistic small-step operational semantics

Dyn: Simple Interpreter for Dynamically Typed Languages

Features

- Operand stack
- Dynamic memory
- *n*-ary operations
- Conditional jumps
- Recursive function calls

Values

```
data dynamic =
  DI int |
  DF float |
  …
```

Dyn: Execution of Small Example Program

Program

```
Celsius = (Fahrenheit - 32) * 5/9
Load fahrenheit
Push (DI 32)
Op Sub
Push (DI 5)
Op Mul
Push (DI 9)
Op Div
Store celsius
```

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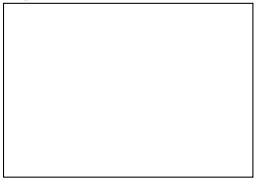
Operand stack



Dynamic memory

{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ $\cdot$}

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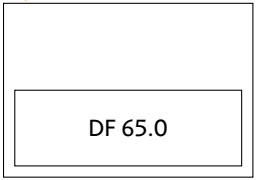

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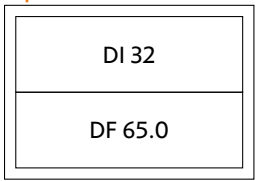

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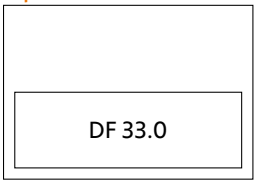

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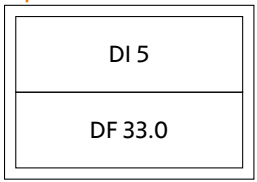

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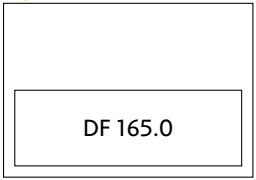

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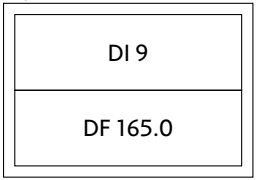

Dynamic memory

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DYN: Execution of Small Example Program

Program

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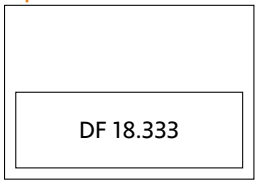
Operand stack



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Operand stack

Dynamic memory

{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ DF 18.333}

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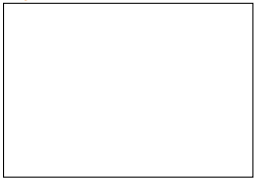

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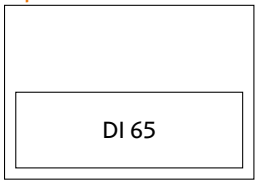

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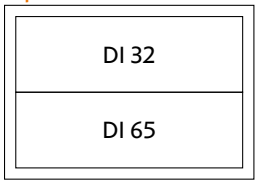

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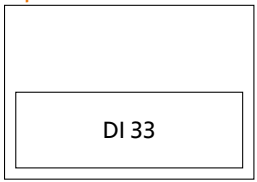

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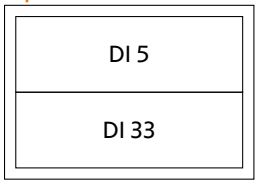

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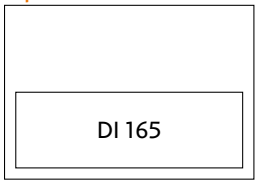

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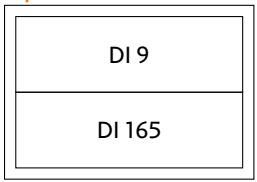

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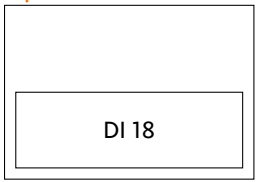

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Store celsius
```

Operand stack

Dynamic memory

{fahrenheit $\mapsto$ DI 65, celsius $\mapsto$ DI 18}

Inlining Reduces the Number of Type Checks in Operations

Example without inlining

```
fun eval Div (DI x) (DI y) = DI (Int.div x y)
  | eval Div (DF x) (DF y) = DF (Float.div x y)
  | eval Div (DI x) (DF y) = DF (Float.div (Float.from_int x) y)
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  | …
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e.g. `eval Div (DF 165.0) (DI 9)`

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Example with inlining

```
fun eval_inl Div_float×int (DF x) (DI y) = DF (Float.div x (Float.from_int y))
  | eval_inl Div_int×int   (DI x) (DI y) = DI (Int.div x y)
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e.g. `eval` $\mathtt{Div}_{\mathtt{float}\times\mathtt{int}}$ `(DF 165.0) (DI 9)`

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e.g. `eval` $\mathtt{Div}_{\mathtt{float}\times\mathtt{int}}$ `(DF 165.0) (DI 9)`

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e.g. `eval Div (DF 165.0) (DI 9)` → 6 type checks

Example with inlining

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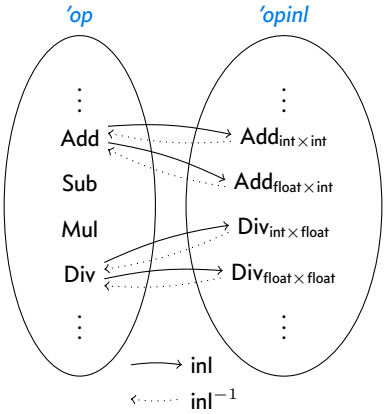
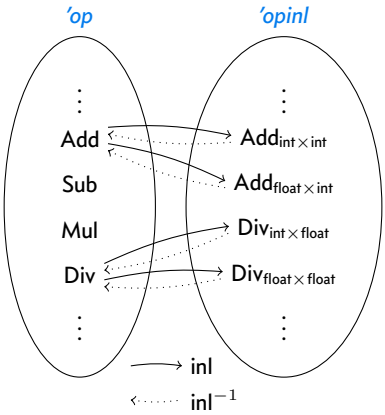
  | …
```

e.g. `eval` $\text{Div}_{\text{float}\times\text{int}}$ `(DF 165.0) (DI 9)` → 2 type checks

The Sets of Regular and Inlined Operations

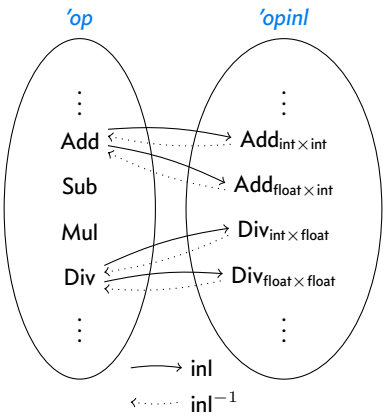


The Sets of Regular and Inlined Operations



locale operations =
  fixes
    arith :: *'op* ⇒ nat **and**
    eval :: *'op* ⇒ dynamic list ⇒ dynamic

The Sets of Regular and Inlined Operations



locale operations =
  fixes
    arith :: *'op* ⇒ nat **and**
    eval :: *'op* ⇒ dynamic list ⇒ dynamic

locale operations-inl = operations +
  fixes
    eval-inl :: *'opinl* ⇒ dynamic list ⇒ dynamic **and**
    inl :: *'op* ⇒ dynamic list ⇒ *'opinl* option **and**
    inl^{-1} :: *'opinl* ⇒ *'op* **and**
    …
  assumes
    inl *op args* = Some op_{inl} ⟹ eval *op* = eval-inl op_{inl} **and**
    inl *op args* = Some op_{inl} ⟹ inl^{-1} op_{inl} = *op* **and**
    …

Inca: Dyn Plus Inline Caching

+1 instruction for inlined operations

Dyn rules

$\vdots$

$\rightarrow_{\text{Dyn}}$-Op

$\vdots$

Inca rules

$\vdots$

$\rightarrow_{\text{Inca}}$-Op-Inl

$\rightarrow_{\text{Inca}}$-Op-No-Inl

$\rightarrow_{\text{Inca}}$-Op-Cache-Hit

$\rightarrow_{\text{Inca}}$-Op-Cache-Miss

$\vdots$

INCA: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
→  Load fahrenheit
   Push (DI 32)
   Op Sub
   Push (DI 5)
   Op Mul
   Push (DI 9)
   Op Div
   Store celsius
```

Operand stack

Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Language	Pass	Type checks
DYN	all	0
INCA	1^{st}	0
INCA	n^{th}	

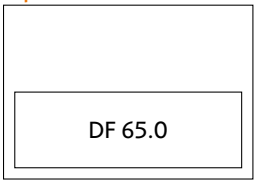
INCA: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
→  Push (DI 32)
   Op Sub
   Push (DI 5)
   Op Mul
   Push (DI 9)
   Op Div
   Store celsius
```

Language	Pass	Type checks
DYN	all	0
INCA	1^{st}	0
INCA	n^{th}	

Operand stack



Dynamic memory

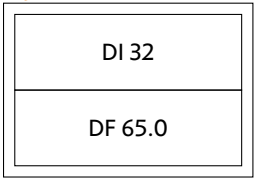
{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ ·}

Inca: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
→  Op Sub
   Push (DI 5)
   Op Mul
   Push (DI 9)
   Op Div
   Store celsius
```

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Language	Pass	Type checks
Dyn	all	0
Inca	1^{st}	0
Inca	n^{th}	

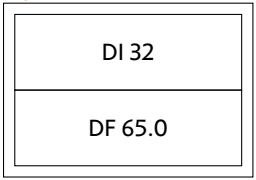
INCA: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
→  OpInl Sub_float×int
   Push (DI 5)
   Op Mul
   Push (DI 9)
   Op Div
   Store celsius
```

Language	Pass	Type checks
DYN	all	6
INCA	1^{st}	8
INCA	n^{th}	

Operand stack



Dynamic memory

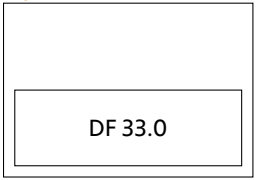
{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ ·}

INCA: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
→  Push (DI 5)
   Op Mul
   Push (DI 9)
   Op Div
   Store celsius
```

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Language	Pass	Type checks
DYN	all	6
INCA	1^{st}	8
INCA	n^{th}	

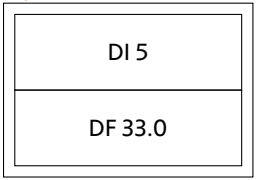
INCA: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
→  Op Mul
   Push (DI 9)
   Op Div
   Store celsius
```

Language	Pass	Type checks
DYN	all	6
INCA	1^{st}	8
INCA	n^{th}	

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

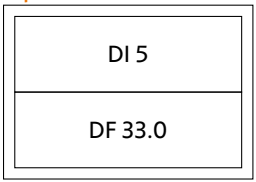
Inca: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
→  OpInl Mul_float×int
   Push (DI 9)
   Op Div
   Store celsius
```

Language	Pass	Type checks
Dyn	all	12
Inca	1^{st}	16
Inca	n^{th}	

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

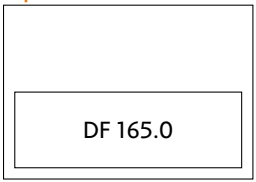
INCA: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
→  Push (DI 9)
   Op Div
   Store celsius
```

Language	Pass	Type checks
DYN	all	12
INCA	1^{st}	16
INCA	n^{th}	

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

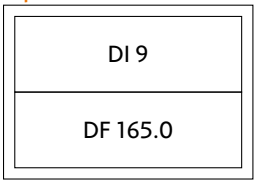
Inca: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
→  Op Div
   Store celsius
```

Language	Pass	Type checks
Dyn	all	12
Inca	1^{st}	16
Inca	n^{th}	

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

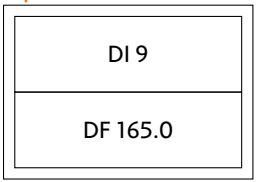
Inca: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
→  OpInl Div_float×int
   Store celsius
```

Language	Pass	Type checks
Dyn	all	18
Inca	1^{st}	24
Inca	n^{th}	

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

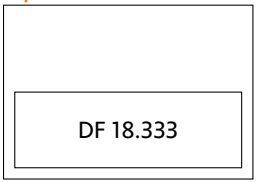
INCA: Execution of Small Example Program

Program: 1st pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
→  Store celsius
```

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Inca: Execution of Small Example Program

Program: 1st pass

```
Celsius = (Fahrenheit - 32) * 5/9
Load fahrenheit
Push (DI 32)
OpInl Sub_float×int
Push (DI 5)
OpInl Mul_float×int
Push (DI 9)
OpInl Div_float×int
Store celsius
```

Language	Pass	Type checks
Dyn	all	18
Inca	1^{st}	24
Inca	n^{th}	

Operand stack

Dynamic memory

{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ DF 18.333}

INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
→  Load fahrenheit
   Push (DI 32)
   OpInl Sub_{float×int}
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack

Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

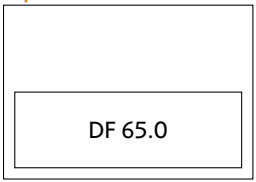
Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	0

INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
→  Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	0

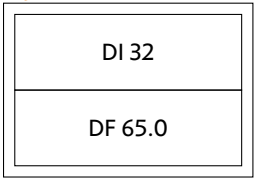
INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
→  OpInl Sub_{float×int}
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	0

Operand stack



Dynamic memory

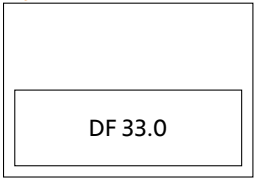
{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ ·}

INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
→  Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	4

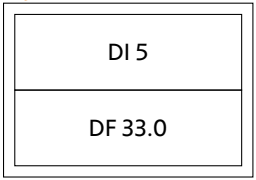
INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
→  OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	4

Operand stack



Dynamic memory

{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ ·}

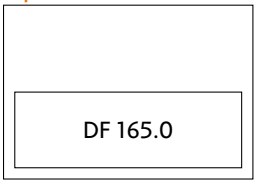
INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
→  Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	8

Operand stack



Dynamic memory

{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ ·}

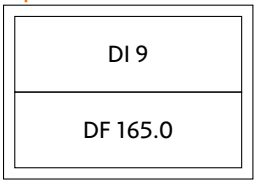
INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
→  OpInl Div_float×int
   Store celsius
```

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	8

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

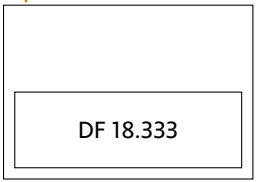
INCA: Execution of Small Example Program

Program: n^{th} pass

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
→  Store celsius
```

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	12

Operand stack



Dynamic memory

{fahrenheit ↦ DF 65.0, celsius ↦ ·}

INCA: Execution of Small Example Program

Program: n^{th} pass

```
Celsius = (Fahrenheit - 32) * 5/9
Load fahrenheit
Push (DI 32)
OpInl Sub_float×int
Push (DI 5)
OpInl Mul_float×int
Push (DI 9)
OpInl Div_float×int
Store celsius
```

Language	Pass	Type checks
DYN	all	18
INCA	1^{st}	24
INCA	n^{th}	12

Operand stack

Dynamic memory

{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ DF 18.333}

Inca: Compiler is Sound and Complete

The compiler $\mathcal{C}$ trivialy lifts instructions from Dyn to Inca.

Soundness

There exists a bisimulation $\sim$, such that $p \sim \mathcal{C}\, p$.

Completeness

$\mathcal{C}$ is a total function.

Unboxing Completely Removes Type Checks in Operations

Example without unboxing

```
fun eval_inl Div_{float×int}   (DF x) (DI y) = DF (Float.div x (Float.from_int y))
  | eval_inl Div_{int×int}     (DI x) (DI y) = DI (Int.div x y)
  | eval_inl Div_{float×float} (DF x) (DF y) = DF (Float.div x y)
  | eval_inl Div_{int×int}     (DI x) (DF y) = DF (Float.div (Float.from_int x) y)
  | …
```

Unboxing Completely Removes Type Checks in Operations

Example without unboxing

```
fun eval_inl Div_float×int   (DF x) (DI y) = DF (Float.div x (Float.from_int y))
  | eval_inl Div_int×int     (DI x) (DI y) = DI (Int.div x y)
  | eval_inl Div_float×float (DF x) (DF y) = DF (Float.div x y)
  | eval_inl Div_int×int     (DI x) (DF y) = DF (Float.div (Float.from_int x) y)
  | …
```

Example with unboxing

```
fun eval_ubx Div_float×int x y = Float.div x (Float.from_int y)
  | …
```

UBX: INCA Plus Unboxing

+n instructions for pushing unboxed values
+2 instructions for loading/storing unboxed values
+1 instructions for unboxed operations

INCA rules

⋮

$\rightarrow_{\text{INCA}}$-Push

$\rightarrow_{\text{INCA}}$-Load

$\rightarrow_{\text{INCA}}$-Store

⋮

UBX rules

⋮

$\rightarrow_{\text{UBX}}$-Push

$\rightarrow_{\text{UBX}}$-Push-Int

$\rightarrow_{\text{UBX}}$-Push-Float

$\rightarrow_{\text{UBX}}$-Load

$\rightarrow_{\text{UBX}}$-Load-Ubx-Hit

$\rightarrow_{\text{UBX}}$-Load-Ubx-Miss

$\rightarrow_{\text{UBX}}$-Op-Ubx

$\rightarrow_{\text{UBX}}$-Store

$\rightarrow_{\text{UBX}}$-Store-Ubx

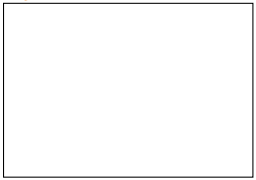
⋮

Ubx: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
→  LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
   PushInt 5
   OpUbx Mul_{float×int}
   PushInt 9
   OpUbx Div_{float×int}
   StoreUbx float celsius
```

Operand stack



Dynamic memory

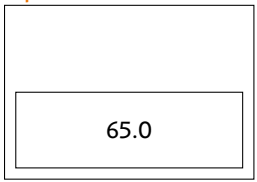
{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ ·}

Ubx: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
→  PushInt 32
   OpUbx Sub_float×int
   PushInt 5
   OpUbx Mul_float×int
   PushInt 9
   OpUbx Div_float×int
   StoreUbx float celsius
```

Operand stack



Dynamic memory

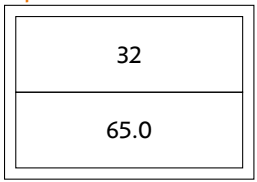
{fahrenheit ↦ DF 65.0, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
→  OpUbx Sub_float×int
   PushInt 5
   OpUbx Mul_float×int
   PushInt 9
   OpUbx Div_float×int
   StoreUbx float celsius
```

Operand stack



Dynamic memory

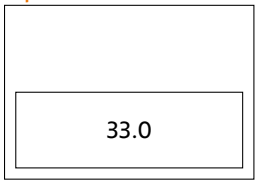
{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ ·}

UBX: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_float×int
→  PushInt 5
   OpUbx Mul_float×int
   PushInt 9
   OpUbx Div_float×int
   StoreUbx float celsius
```

Operand stack



Dynamic memory

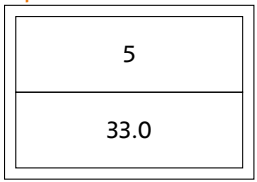
{fahrenheit ↦ DF 65.0, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_float×int
   PushInt 5
→  OpUbx Mul_float×int
   PushInt 9
   OpUbx Div_float×int
   StoreUbx float celsius
```

Operand stack



Dynamic memory

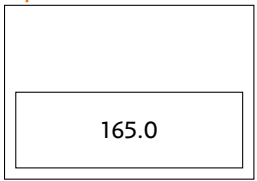
{fahrenheit ↦ DF 65.0, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_float×int
   PushInt 5
   OpUbx Mul_float×int
→  PushInt 9
   OpUbx Div_float×int
   StoreUbx float celsius
```

Operand stack



Dynamic memory

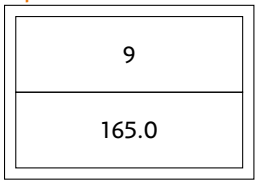
{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Ubx: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_float×int
   PushInt 5
   OpUbx Mul_float×int
   PushInt 9
→  OpUbx Div_float×int
   StoreUbx float celsius
```

Operand stack



Dynamic memory

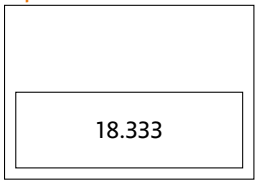
{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Ubx: Execution of Small Example Program

Program with **cache hit**

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_float×int
   PushInt 5
   OpUbx Mul_float×int
   PushInt 9
   OpUbx Div_float×int
→  StoreUbx float celsius
```

Operand stack



Dynamic memory

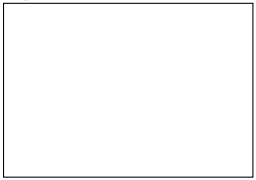
{fahrenheit ↦ DF 65.0, celsius ↦ ·}

Ubx: Execution of Small Example Program

Program with **cache hit**

```
Celsius = (Fahrenheit - 32) * 5/9
LoadUbx float fahrenheit
PushInt 32
OpUbx Sub_float×int
PushInt 5
OpUbx Mul_float×int
PushInt 9
OpUbx Div_float×int
StoreUbx float celsius
```

Operand stack



Dynamic memory

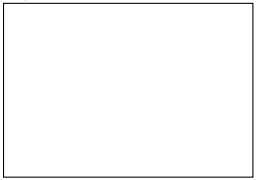
{fahrenheit $\mapsto$ DF 65.0, celsius $\mapsto$ DF 18.333}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
→  LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_float×int
   PushInt 5
   OpUbx Mul_float×int
   PushInt 9
   OpUbx Div_float×int
   StoreUbx float celsius
```

Operand stack



Dynamic memory

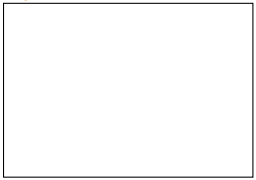
{fahrenheit $\mapsto$ DI 65, celsius $\mapsto$ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
→  Load fahrenheit
   Push (DI 32)
   OpInl Sub_{float×int}
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Dynamic memory

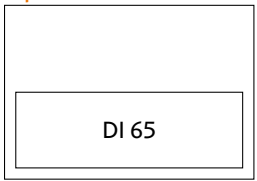
{fahrenheit ↦ DI 65, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
→  Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

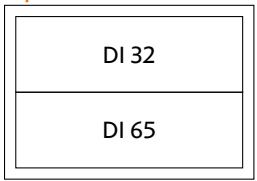
{fahrenheit ↦ DI 65, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
→  OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

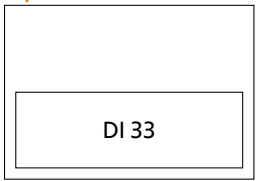
{fahrenheit $\mapsto$ DI 65, celsius $\mapsto$ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
→  Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

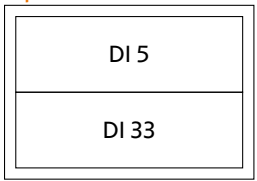
{fahrenheit ↦ DI 65, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
→  OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

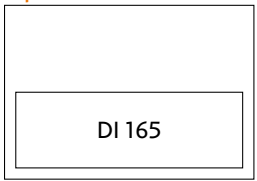
{fahrenheit ↦ DI 65, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
→  Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

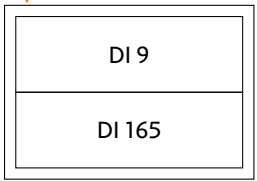
{fahrenheit ↦ DI 65, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
→  OpInl Div_float×int
   Store celsius
```

Operand stack



Dynamic memory

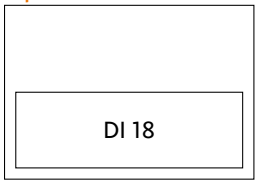
{fahrenheit ↦ DI 65, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
   Celsius = (Fahrenheit - 32) * 5/9
   Load fahrenheit
   Push (DI 32)
   OpInl Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
→  Store celsius
```

Operand stack



Dynamic memory

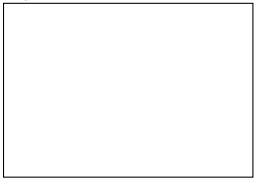
{fahrenheit ↦ DI 65, celsius ↦ ·}

UBX: Execution of Small Example Program

Program with **cache miss**

```
Celsius = (Fahrenheit - 32) * 5/9
Load fahrenheit
Push (DI 32)
OpInl Sub_float×int
Push (DI 5)
OpInl Mul_float×int
Push (DI 9)
OpInl Div_float×int
Store celsius
```

Operand stack



Dynamic memory

{fahrenheit $\mapsto$ DI 65, celsius $\mapsto$ DI 18}

Ubx: Static Optimization

Abstract interpretation

Ubx: Static Optimization

Abstract interpretation

Static type information

- Constants
- Function parameters
- Static analysis

Oracle

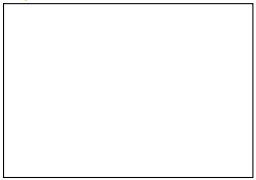
What is the exected type when loading a value from memory?

Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
→  Load fahrenheit
   Push (DI 32)
   OpInl Sub_{float×int}
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

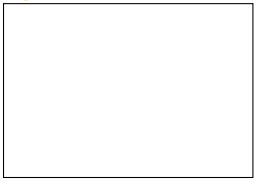
$\{\text{fahrenheit} \mapsto \mathit{float}, \mathit{celsius} \mapsto \cdot\}$

Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
→  LoadUbx float fahrenheit
   Push (DI 32)
   OpInl Sub_{float×int}
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

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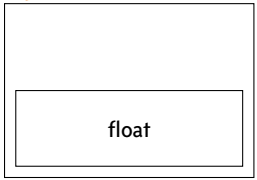
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
→  Push (DI 32)
   OpInl Sub_{float×int}
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

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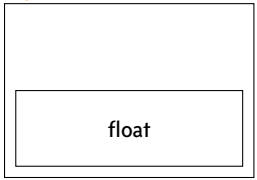
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
→  PushInt 32
   OpInl Sub_{float×int}
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

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Ubx: Example of Static Optimization

Program

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   PushInt 32
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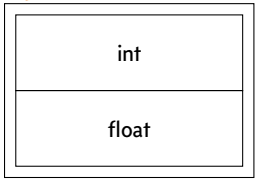
   Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

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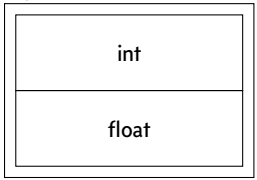
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
→  OpUbx Sub_float×int
   Push (DI 5)
   OpInl Mul_float×int
   Push (DI 9)
   OpInl Div_float×int
   Store celsius
```

Operand stack



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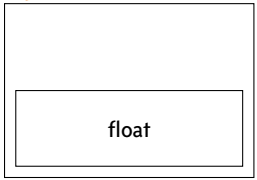
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
→  Push (DI 5)
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

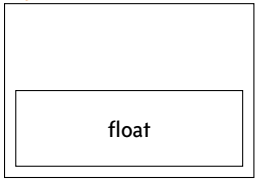
$\{\text{fahrenheit} \mapsto \mathit{float}, \mathit{celsius} \mapsto \cdot\}$

Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
→  PushInt 5
   OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

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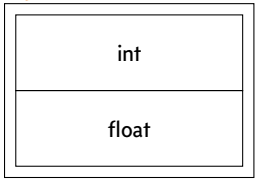
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
   PushInt 5
→  OpInl Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



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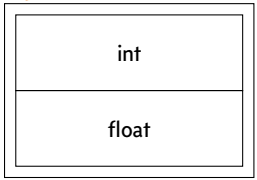
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
   PushInt 5
→  OpUbx Mul_{float×int}
   Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



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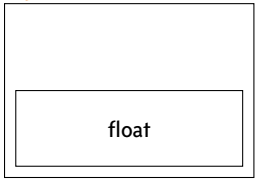
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
   PushInt 5
   OpUbx Mul_{float×int}
→  Push (DI 9)
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

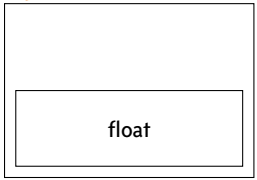
$\{\text{fahrenheit} \mapsto \mathit{float}, \mathit{celsius} \mapsto \cdot\}$

Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
   PushInt 5
   OpUbx Mul_{float×int}
→  PushInt 9
   OpInl Div_{float×int}
   Store celsius
```

Operand stack



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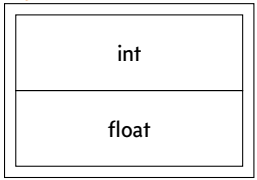
Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
   PushInt 5
   OpUbx Mul_{float×int}
   PushInt 9
→  OpInl Div_{float×int}
   Store celsius
```

Operand stack



Oracle

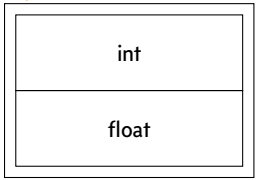
$\{\text{fahrenheit} \mapsto \mathit{float}, \mathit{celsius} \mapsto \cdot\}$

Ubx: Example of Static Optimization

Program

```
   Celsius = (Fahrenheit - 32) * 5/9
   LoadUbx float fahrenheit
   PushInt 32
   OpUbx Sub_{float×int}
   PushInt 5
   OpUbx Mul_{float×int}
   PushInt 9
→  OpUbx Div_{float×int}
   Store celsius
```

Operand stack



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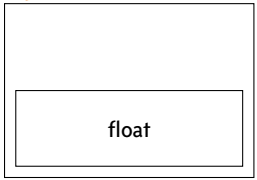

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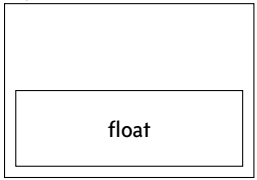

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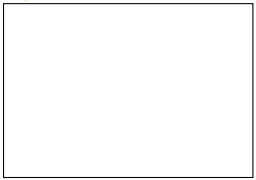

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UBX: Compiler is Sound and Almost Complete

The compiler $\mathcal{C}$

1. lifts instructions from INCA to UBX;
2. performs a single-pass, optimizing abstract analysis;
3. falls back to the unoptimized version if optimization failed.

Soundness

If p is "well-typed", there exists a bisimulation $\sim$, such that $p \sim \mathcal{C}\, p$.

Completeness

$\mathcal{C}$ is a partial function.
It is complete only for "well-typed" programs.

Summary

Specification of purely interpretative optimizations

Dyn: *baseline*

```
Load fahrenheit
Push (DI 32)
Op Sub
Push (DI 5)
Op Mul
Push (DI 9)
Op Div
Store celsius
```

Inca: *inline caching*

```
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Push (DI 5)
OpInl Mul_float×int
Push (DI 9)
OpInl Div_float×int
Store celsius
```

Ubx: *unboxing*

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PushInt 32
OpUbx Sub_float×int
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Complete mechanical formalization

- Realistic small-step operational semantics
- Multiple details kept abstract with locales
- Available in the *Archive of Formal Proofs*
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Thank you